University of Mississippi
13th Annual High School Mathematics Contest
Team Competition (February 10, 2018)

1. Today, Alex is three times as old as Bob was when Alex was as old as Bob is today. When Bob will be as old as Alex is today, the sum of their ages will be 70 years. How old is each of them today?

2. Find integer numbers m and n such that $\sqrt{m} - \sqrt{n} = 2\sqrt{2 - \sqrt{3}}$.

3. Find the maximum value of the product $x^2 y^2 (3 - 2x^2 - y^2)$.

4. Show that for all $p, q \geq 1$,
$$p\sqrt{q - 1} + q\sqrt{p - 1} \leq pq.$$

5. There are two bags. The first bag contains two white balls and one black ball, while the second bag contains one white ball and two black balls. We randomly draw a ball from the first bag and put it into the second bag. Now if we draw a ball from the second bag, what is the probability that the ball drawn is white?

6. (a) How many 4 digit numbers are there such that the first digit is odd, the last digit is even, and all four digits are different?

   (b) How many 4 digit numbers are there such that the first digit is even, the last digit is odd, and all four digits are different?

7. If a real-valued function of one real variable $f : \mathbf{R} \to \mathbf{R}$ satisfies $f(f(x)) = x + 1$, show that for every integer n, $f(n)$ is not an integer.

8. We write 2018 as a sum of positive integers such that their product is biggest possible. Find this product.

9. Show that it is possible to decompose two identical regular hexagons into a total of six pieces such that they can be rearranged to form an equilateral triangle with no two pieces overlapping. (Your answer can be in the form of a drawing showing the decomposition of the hexagons.)

10. In an a town Squareville the streets form a square grid (all the streets intersect at a 90 degree angle, and the neighboring parallel streets are equidistant). A car starts driving from a point A, with constant speed. Every 15 seconds, the car turns at a 90 degree angle, to the left or to the right. Prove that the car can come back to point A only after a full number of minutes.