1. Solve the following system of equations for x, y, z:

$$\log_2 x + \log_4 y + \log_4 z = 2$$
$$\log_3 y + \log_9 z + \log_9 x = 2$$
$$\log_4 z + \log_{16} x + \log_{16} y = 2$$

2. Find the remainder of

$$10^{5^{10^{5^{10}}}} + 5^{10^{5^{10^5}}}$$

   when you divide it by 11.

3. There are 10 volleyball teams competing in a round-robin tournament, where each team plays each other team. No game is allowed to end in a tie. Let x_1 be the number of wins of the first team and y_1 the number of losses of the first team. Let x_2 be the number of wins of the second team, and y_2 the number of losses, and so forth, with x_i being the number of wins of the ith team and y_i being the number of losses of the ith team.

   Show that
   $$x_1^2 + x_2^2 + x_3^2 + \cdots + x_9^2 + x_{10}^2 = y_1^2 + y_2^2 + y_3^2 + \cdots + y_9^2 + y_{10}^2$$

4. On a certain island, there are two types of fairies: good fairies and bad fairies. The good fairies always tell the truth and the bad fairies always lie.

   A shipwrecked sailor knows all this. He meets two fairies on the island, A and B. The two fairies A and B know each other (and whether they are good or bad) and only answer questions with "yes" or "no".

   (a) First, he asks A, "Are you both bad fairies?" What possible answer would allow him to know what kinds of fairies both A and B are?

   (b) He gets an answer that doesn't allow him to conclude. He now asks A, "Are you different kinds of fairies?" What answer does he get if he now knows what kinds of fairies they are?

5. A, B, C are positive integers that satisfy

$$A^2 + B^2 = C^2,$$

   and A is prime. Show that B must be larger than A.

6. We have a field of grass in which grass grows at a constant rate. 60 cows eat all the grass in the field in 24 days. 30 cows eat all the grass in 60 days.

   (a) How many cows are needed to eat all the grass in 100 days?

   (b) In how many days will 10 cows eat all the grass in the field?

7. Find all functions $f : \mathbb{R} \to \mathbb{R}$ with the property that

$$f(x - y) = f(x) + f(y) - 2xy, \quad \text{for all real } x, y.$$

8. There is a shelf with 14 books on it. In how many ways can you choose 5 books so that no two of the chosen books are next to each other on the shelf?

9. A group of 60 delegates attend a meeting. Each one speaks at least one of the following languages: English, French and Spanish. 19 delegates speak only English. 10 delegates speak only Spanish and 11 speak only French. 11 speak both French and English. 10 speak both Spanish and English. 7 speak both Spanish and French. How many speak all 3 languages?

10. What is the maximal number of subsets of $\{1, 2, \ldots, 100\}$ that we can choose so that the union of any two of them is the whole set $\{1, 2, \ldots, 100\}$?