University of Mississippi

Individual Competition

12^{th} Annual High School Mathematics Contest

1. In the morning, Al grabs socks from his dresser drawer without looking at them. In his dresser, he has 16 black socks and 10 blue socks. How many does he need to take to be sure to have a matching pair?

(a) 2
(b) 3
(c) 11
(d) 17
(e) 26

2. Estelle needs to get to the airport at 10 AM to catch her flight. She leaves the house at 9 AM. She starts driving at 40 miles per hour. When she has driven half the distance, she realizes she needs to speed up: she drives the second half of the way at 60 miles per hour. She arrives exactly at 10 AM. How far is her house from the airport, in miles?

(a) 43
(b) 45
(c) 48
(d) 50
(e) 52

3. January 1st, 2017 occurred on a Sunday. January 1st, 2025 will be on what day of the week?

(a) Monday
(b) Tuesday
(c) Wednesday
(d) Thursday
(e) Friday

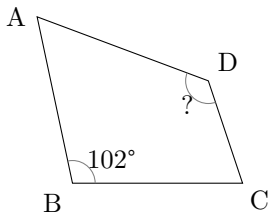
4. The town of Albion is west of Bethany. Chester is east of Albion but west of Bethany. Dexter is east of Endfield, but west of Chester and Albion. Of the five towns listed, find the one that is farthest west.

(a) Albion
(b) Bethany
(c) Chester
(d) Dexter
(e) Endfield

5. The sum of two numbers is $x + y = 9$ and their product is $xy = 5$. What is the sum of their squares $x^2 + y^2$?

(a) 45
(b) 53
(c) 71
(d) 76
(e) 81

6. In the quadrilateral ABCD (see diagram), the angle $\angle ABC$ measures 102° and the lengths $|AB| = |BC| = |BD|$. What does the angle $\angle ADC$ measure in degrees?



(a) 39
(b) 129
(c) 78
(d) 60
(e) 45

7. A rectangular block of dimensions 5x4x3 has its surface painted blue. We then cut it up in to 1x1x1 cubes. How many of the cubes have **exactly** one face painted blue?

(a) 11
(b) 22
(c) 52
(d) 60
(e) 94

8. Anne has five red cards numbered 1 through 5 and four blue cards numbered 3 through 6. She stacks the cards so that the colors alternate and so that the number on each red card is a divisor of the number on each neighboring blue card. What is the sum of the numbers on the middle three cards?

(a) 8
(b) 9
(c) 10
(d) 11
(e) 12

9. It is known that

$$35! = 10333147966386144929X6665133752320000000$$

where one digit has been replaced by X. (Remember that $n!$ is the product of the first n positive integers, i.e $n! = 1 \cdot 2 \cdot 3 \cdot \ldots (n-1) \cdot n$.) What is the digit that has been replaced by X?

(a) 0
(b) 2
(c) 3
(d) 6
(e) 9

10. It is known for some positive integer a that exactly 3 of the 4 following statements are correct:

(i) 2 divides a

(ii) 4 divides a

(iii) 12 divides a

(iv) 24 divides a

Which of these statements is the incorrect one?
(a) (i)
(b) (ii)
(c) (iii)
(d) (iv)
(e) Not enough information

11. What is the largest integer that is a divisor of

$$(n+1)(n+3)(n+5)(n+7)(n+9)$$

for all positive even integers n?
(a) 3
(b) 5
(c) 11
(d) 15
(e) 165

12. Crystal is thinking of a 4-digit number. She gives you the following four clues:

(i) All of its digits are distinct.

(ii) The number is a multiple of 11.

(iii) The sum of the first and last digit is 3 more than a multiple of 11.

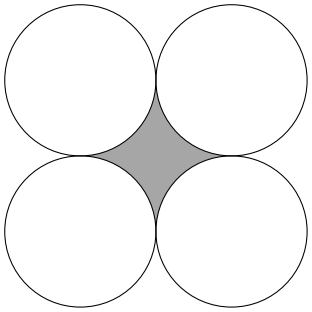
(iv) My number is the SMALLEST number that satisfies all these conditions.

What are the last two digits of her number?
(a) 12
(b) 21
(c) 38
(d) 42
(e) 56

13. If m, n and 1 are non-zero roots of the cubic equation $x^3 - mx^2 + nx - 1 = 0$, then the sum of the roots is:
(a) -1
(b) 0
(c) 1
(d) 2
(e) 3

14. Greg the Gambler starts with k dollars. He pays \$ 10 admission to enter a casino. While there, he doubles his money he entered with. He leaves and pays \$ 10 for parking. He then pays \$ 10 to enter another casino, and again doubles the money he entered with. When he leaves, he pays \$ 10 for parking again, and has no money left. What is k?
(a) \$ 17.25
(b) \$ 20
(c) \$ 22.50
(d) \$ 25
(e) \$ 30

15. Four circles of radius 10 cm are arranged as indicated in the diagram. Find the area of the shaded region, in square centimeters, rounded to the nearest whole number.



(a) 78
(b) 86
(c) 214
(d) 314
(e) 400

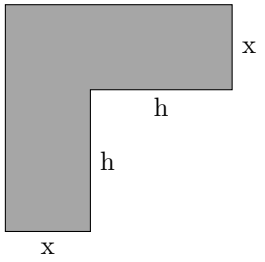
16. 20 lines are drawn in the plane so that no two are parallel and so no three meet in a single point. They divide the plane in to 211 non-overlapping regions. Some of these regions have finite area and others have infinite area. How many of the regions have finite area?
(a) 40
(b) 81
(c) 121
(d) 171
(e) 191

17. If $f(x-y) = f(x)f(y)$ for every two real numbers x, y, and $f(0) \neq 0$, what is $f(3)$ equal to?
(a) -3
(b) -1
(c) 0
(d) 1
(e) 3

18. At a certain high school, there are three extracurricular sports teams: Rugby, Curling and Tiddlywinks. Each of the three teams has 15 students on it. There are 5 students who play on both the Rugby and Curling teams. There are 8 students who do Curling and Tiddlywinks. There are 3 students who play Rugby and Tiddlywinks. We are told there are 31 students who participate in at least one of these sports. How many students are on all three teams?

(a) 0
(b) 1
(c) 2
(d) 5
(e) 8

19. In the diagram, all lines that meet do so at right angles. If the area of the figure is 30 square centimeters, and $2 < x < 3$, which inequality must hold for h?



(a) $3.5 < h < 6.5$
(b) $4 < h < 7$
(c) $2 < h < 6$
(d) $2.5 < h < 5.5$
(e) $4 < h < 8$

20. What is the sum of all numbers obtained by permuting the digits of 516?
(a) 2448
(b) 2664
(c) 2768
(d) 2862
(e) 2868

21. Four red points, three blue points and two green points are painted on a plane, with no three points colinear. How many different triangles can be made, connecting these points, so that each triangle's vertices are of exactly two colors?
(a) 9
(b) 24
(c) 30
(d) 55
(e) None of these

22. Samantha has a mixture of water and salt that is 1% salt by mass. She takes 1 kg of the mixture and boils it down until it is 2% salt by mass. How much water did she boil away?
(a) 0.01 kg
(b) 0.02 kg
(c) 0.1 kg
(d) 0.2 kg
(e) 0.5 kg

23. Viktor has a vase that contains a black ball or a white ball, with equal probability. He then puts a white ball in the vase. You pull a ball out, and it turns out to be white. What is the probability that the ball remaining in the vase is white?
(a) 1/3
(b) 1/2
(c) 1/4
(d) 3/4
(e) 2/3