1. If \( a + b + c = 1 \) with \( a, b, c \geq 0 \), show that \( \sqrt{a} + \sqrt{b} + \sqrt{c} \leq \sqrt{3} \).

2. Compute the sum
\[
S_n = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)}.
\]

3. Consider any three consecutive positive integers. Show that the cube of the largest cannot be the sum of the cubes of the other two.

4. What is the largest number by which \( n^3 - n \) is divisible for all possible integers \( n \)?

5. What is the last digit of \( 3^{2014} - 2^{2014} \)?

6. Suppose that
\[
\sin \alpha + \sin \beta = m \\
\cos \alpha + \cos \beta = n \quad \text{with} \quad m^2 + n^2 \neq 0.
\]
(a) Find \( \cos(\alpha - \beta) \).
(b) Find \( \sin(\alpha + \beta) \).

7. Let \( a, b \) and \( c \) be real numbers satisfying the inequalities:
\[
|a| \leq |b - c|, \quad |b| \leq |c - a|, \quad |c| \leq |a - b|.
\]
Show that one of these numbers is the difference of the other two.

8. How many seating orderings can one make for 6 boys and 3 girls, if no two girls are to sit next to each other and they are sitting
(a) in a row.
(b) at a round table. (A seating arrangement obtained by rotating another one is considered to be the same ordering.)

9. There are 100 students at a high school who play at least one of the following sports: Tennis, Badminton and Ping-pong. 28 play only tennis, 30 only Badminton and 20 only Ping-pong. 11 students play Badminton and Ping-pong, 9 students play Tennis and Ping-pong and 8 students Tennis and Badminton. How many students play all three sports?

10. Given that numbers \( x, y, z \geq 0 \) satisfying \( x^2 + y^2 + z^2 \leq 1 \), what is the probability that \( x + y + z \leq 1 \)?