University of Mississippi

2nd Annual High School Mathematics Contest
Team Competition
October 21, 2006

1. For real numbers r, let $\lfloor r \rfloor$ be the greatest integer that is less than or equal to r. Solve the inequality

$$\lfloor x \rfloor + \lfloor x+3 \rfloor \leq 17$$

2. Find the minimum distance from the point whose cartesian coordinates in space are $(8, 12, -9)$ to some point on a unit sphere centered at the origin. Be sure to prove your answer.

3. Rationalize the denominator and simplify:

$$\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$$

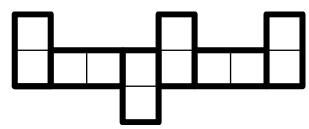
4. What is the maximum number of bishops that may be placed on an 8×8 chess board without any bishop being in a square threatened by any other bishop? (A bishop threatens any square that lies on the same diagonal as the bishop).

5. If both a and b are strictly between 0 and 1, prove that $a+b-ab$ must be strictly between 0 and 1.

6. A certain pattern is used to generate a sequence of numbers, the first few of which are shown below. Prove that no matter how far this sequence is continued, no 4 will ever appear as a digit.

$$1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211$$

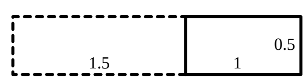
7. Dominoes of dimension $2'' \times 1''$ are used to cover a $1'' \times 8''$ strip of $1'' \times 1''$ squares. Dominoes may be placed either along the strip so as to cover two of the squares, or they may be placed so that one square inch of the domino lies above or below the strip. One such covering for a strip for which is shown. Find the number of ways there are to cover the strip. (Coverings that differ by a rotation are considered distinct.)



8. Evaluate

$$\frac{\sum_{k=0}^{20} \cos\left(\frac{\pi(k-5)}{20}\right)}{\sum_{k=0}^{20} \sin\left(\frac{\pi k}{20}\right)}$$

9. A flat, triangular parking lot has dimensions 15 car lengths by 20 car lengths by 25 car lengths, where a "car length" is a fixed unit of measure that is a little longer than the average car's length. A parking space must be rectangular, with dimensions one car length long by one-half car length wide, and every space must have 1.5 car lengths of clearance behind it (area within the parking lot where there is no parking space). Different parking spaces may share the same clearance space, and every spot must be accessible from some point on the perimeter of the lot, not counting the curb that bounds that parking space itself. For simplicity, assume that the long side of every space is perpendicular to the 15-car-length side of the lot. Determine the maximum number of parking spaces the lot can hold.



10. Let $[\mathscr{F}]$ represent the area of figure $\mathscr{F}$. If $ABCD$ is a trapezoid with $AD \parallel BC$ and O is the intersection of the trapezoid's diagonals, prove that

$$\frac{[AOB]}{[ABCD]} < \frac{1}{4}$$