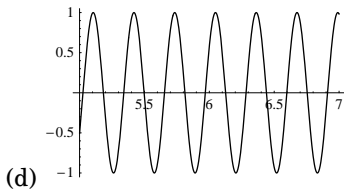
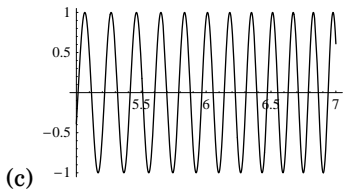
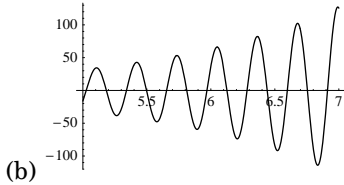
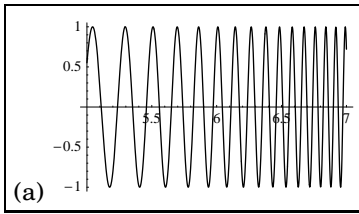


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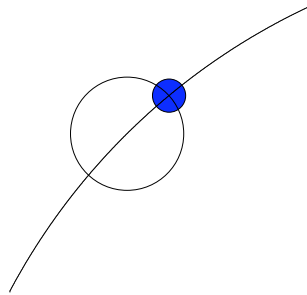
2nd Annual High School Mathematics Contest
Individual Competition Answer Key
October 21, 2006

- Beginning with your feet on the ground, you ascend a ten-rung ladder by going up three rungs, then going down two rungs, going up three, then down two, and so on. Each movement, up or down one rung, takes 0.2 seconds. After how many seconds will your feet reach the top rung?
(a) 7.6
(b) 7.8
(c) 8.0
(d) 8.2
(e) None of the above
- One piece of gum, six ice cream cones, and four sodas cost \$10.68, whereas a piece of gum, four ice cream cones, and six sodas cost \$11.06. What is the positive difference in price between one ice cream cone and one soda?
(a) 17¢
(b) 18¢
(c) 19¢
(d) 20¢
(e) None of the above
- A rotating sphere has energy E given by $E = \frac{1}{2}I\omega^2$, where I is given in terms of the mass M and radius R of the sphere by $I = \frac{2}{5}MR^2$. Solve for ω in terms of E , M , and R .
(a) $\sqrt{\frac{2E}{I}}$
(b) $\sqrt{\frac{5I}{2M}}$
(c) $\sqrt{\frac{5E}{M} \cdot \frac{1}{R}}$
(d) $\frac{5E}{MR^2}$
(e) None of the above
- Which of the following is equal to $9! + 8!$, where $n!$ is defined to be $n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$?
(a) $10!$
(b) $17!$
(c) $\frac{11 \cdot 8!}{9}$
(d) $\frac{10!}{9}$
(e) None of the above
- In the United States, FM radio channels fall in the range 87.9 MHz to 107.9 MHz, with a channel spacing of 0.2 MHz. How many different channels are there?
(a) 100
(b) 200
(c) 201
(d) 202
(e) None of the above (101)
- How many square feet must be in the area of Anderson's living room if his living room furniture takes up 24 ft.² and 60% of his living room floor isn't covered by furniture?
(a) 44
(b) 50
(c) 56
(d) 60
(e) None of the above
- Let x and y be two real numbers that, when expressed in simplest radical form, may be written as $x = a\sqrt{b}$ and $y = c\sqrt{d}$ for natural numbers a , b , c , and d . If xy is an integer, then which of the following statements, if any, must be true?
(a) a and c must both have divisors that are perfect squares
(b) $b = d$
(c) $a^2b = c^2d$
(d) b and d must be divisible by a and c , respectively
(e) None of the above
- Let k and a be fixed constants. The function $P_A(z) = \frac{k}{z}$ is designed to approximate the function $P(z) = \frac{kz}{z^2 - a^2}$ for large z . Which inequality describes exactly the values of z for which $P_A(z)$ approximates $P(z)$ with an error of less than 1%?
(a) $z = 8ak$
(b) $z < 16ak$
(c) $z > 16ak$
(d) $z > 11a$
(e) $z > a\sqrt{101}$
- Find the greatest integer that does not exceed $(10^{12} + 10)^{1/2}$.
(a) 99
(b) 100
(c) 1,000,000
(d) 1,000,001
(e) 1,000,002
- If α is chosen at random from the set $\{3, 6, 9, \dots, 99\}$ and β from the set $\{-99, -96, \dots, -3\}$, then how many different possible values are there for $\alpha + \beta$?
(a) 59
(b) 65
(c) 334
(d) 1089
(e) None of the above

11. Which answer choice is a graph of $y = \sin(2^x)$ over the range $x = 5$ to $x = 7$?



12. Let $M(n)$ represent the number of times in year n that the moon passes through the circle that is the earth's orbit around the sun. Find the mode of the set $\{M(1900), M(1901), M(1902), \dots, M(2005), M(2006)\}$. Assume the period of the moon's orbit is 28 days, and that the orbit of the earth and the moon are circular and coplanar.



- (a) 12
 (b) 24
 (c) 26
 (d) 28
 (e) None of the above
13. Determine which answer choice (if any) may fit correctly in the following blank: If p is a perfect square and $p - 1$ is prime, then _____.
- (a) p must be is greater than 10
 (b) there are infinitely many possibilities for p
 (c) $2p + 1$ must be a perfect square
 (d) there are more than 5 possibilities for p
 (e) None of the above

14. How many four-digit numbers have the property that reversing the digits yields a four-digit number that is divisible by 9? (A number is not considered a four digit number if its leading digit is 0.)
- (a) 899
 (b) 900
 (c) 999
 (d) 1000
 (e) None of the above

15. Define a *nice* set of convex polygons to be a set that can be arranged in the plane in such a way that they fit together to form another convex polygon that has no more sides than the polygon in the set that has the most sides. (For example, a set of four equilateral triangles is nice, whereas a set of three equilateral triangles is not.) What is the smallest possible number of elements for a nice set of polygons that includes a pentagon and at least one other polygon?
- (a) 3
 (b) 4
 (c) 5
 (d) 6
 (e) None of the above (2)

16. Pete orders a 14" pizza and Jonathan orders a 10" pizza of the same thickness. Pete remarks, "I have 2 times as much pizza as you do!" To the nearest percent, what is his percent error?
- (a) 2%
 (b) 41%
 (c) 42%
 (d) 43%
 (e) None of the above

17. Solve for x , express your answer as a decimal, and add up all the digits (e.g. if $x = 8.79$ were the solution, then $8 + 7 + 9 = 24$ would be the answer.)

$$\sqrt{x+1} + \sqrt{x-1} = 10$$

- (a) 8 (25.01)
 (b) 10
 (c) 12
 (d) 14
 (e) None of the above

18. How many perfect squares are factors of $14^{14} \cdot 17^{17}$?
- (a) 72
 (b) 81
 (c) 84
 (d) 576
 (e) 624

19. The equation $100x^{100} - 1 = 0$ has two distinct real solutions $x = r$ and $x = s$. Which of the following is true of rs ?

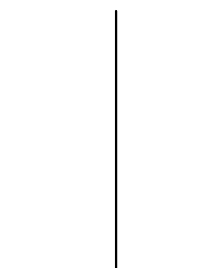
- (a) $rs < -1/4$
- (b) $-1/4 \leq rs < 0$
- (c) $rs = 0$
- (d) $0 < rs \leq 1/4$
- (e) $1/4 < rs$

20. To the nearest hundredth, what is the measure in radians of the smallest positive angle α for which

$$\sin \alpha + \cos \alpha = \frac{1 + \sqrt{3}}{2}$$

- (a) 0.45
- (b) 0.52
- (c) 0.66
- (d) 1.02
- (e) None of the above

21. The cleaning device shown consists of a 12 cm shaft (the vertical segment), and a 10 cm cleaning surface (the horizontal segment). If the device is fixed to a wall at the upper end of the shaft, but is allowed to rotate freely about this point, what is the area of the region on the wall that can be reached by the cleaning surface?

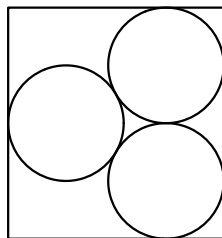


- (a) 25π
- (b) 30π
- (c) 36π
- (d) 44π
- (e) None of the above

22. One hundred unit squares are joined to form a larger square. A single straight line is drawn passing through the interiors of q of the 100 squares. What is the largest possible value of q ?

- (a) 16
- (b) 17
- (c) 18
- (d) 19
- (e) None of the above

23. Three congruent circles are arranged mutually externally tangent as shown, and about them is circumscribed a rectangle with a pair of sides parallel to the line segment connecting the two rightmost circles. To the nearest hundredth, what is the ratio of the longer side length of the rectangle to the



shorter? (The following approximations might be useful: $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$).

- (a) 0.93
- (b) 1.07
- (c) 1.10
- (d) 1.13
- (e) None of the above

24. A bug crawls along the graph $y = \sqrt{10 - x^2}$ from the point $(-\sqrt{10}, 0)$ to the point $(\sqrt{10}, 0)$. He moves at a rate of 10 units per second. His trip will take him t seconds. Which of the following is true?

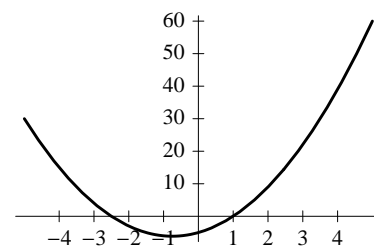
- (a) $0 < t \leq 1$
- (b) $1 < t \leq 1.5$
- (c) $1.5 < t \leq 2$
- (d) $2 < t \leq 2.5$
- (e) None of the above

25. For every integer n , $f(n)$ computes the number of sevens in the prime factorization of n (e.g. $f(98) = 2$). Compute

$$\sum_{n=2}^{1000} f(n)$$

- (a) 163
- (b) 154
- (c) 142
- (d) 133
- (e) None of the above (164)

26. For certain values of the constants a , b , and c , the graph of the equation $y = ax^2 + bx + c$, over the interval $(-5, 5)$ is shown. Which of the following are true?



- i) $a + c = 0$
- ii) $ab > 0$
- iii) $b^2 < 4ac$
- (a) (i) only
- (b) (ii) only
- (c) (i) and (ii)
- (d) (ii) and (iii)
- (e) None of the above

27. What is the maximum value of the absolute value of the determinant of a 3×3 matrix, all of whose entries are 0 or 1?

- (a) 2
- (b) 3
- (c) 4
- (d) 5
- (e) None of the above

28. Compute $\frac{10^{18} + 10^3}{1,000,010}$

(a) 999999000900

(b) 999990000100

(c) 999900000900

(d) 999900000100

(e) None of the above

29. Denote the side lengths of triangle $\triangle ABC$ by a , b , and c where the side of length a is across from vertex A , and that of length b is across from B . Given that $\sin A \sin B = \sin(A + B)$, which of the following gives a formula for the area of $\triangle ABC$?

(a) $ab \sin C$

(b) $c^2/2$

(c) $abc/4(a + b + c)$

(d) $ab \cos C/2$

(e) None of the above

30. What is the smallest value that the function $f(a, b) = \sqrt{4 + a^2} + \sqrt{4 + (a - b)^2} + \sqrt{1 + (14 - b)^2}$ takes on for real values for a and b ?

(a) 12

(b) 13

(c) $\sqrt{5} + 2\sqrt{2} + \sqrt{122}$

(d) $\sqrt{219}$

(e) $\sqrt{221}$