University of Mississippi

3rd Annual High School Mathematics Contest
Individual Competition
October 27, 2007

1. On which birthday does the sixteenth year of one's life begin?

(a) Thirteenth
(b) Fifteenth
(c) Seventeenth
(d) Nineteenth
(e) Not enough information

2. Simplify

$$-9-7-5-3-1+2+4+6+8+10$$

(a) 55
(b) 35
(c) 15
(d) 5
(e) None of the above

3. Which of the following is equivalent to $\frac{7x+14}{7}-x$

(a) 2
(b) 14
(c) x
(d) $6x+2$
(e) None of the above

4. At a basketball game between the Badgers and the Tigers, Jershuntas can only see the units digits of the scores of the two teams. He hears that one of the teams is winning by 8 points, and he can see that the Badgers' score ends in a 4, and the Tigers' score ends in a 2. Based on this information, who is winning?

(a) the Badgers
(b) the Tigers
(c) Either team could be winning.
(d) The teams are tied.
(e) None of the above

5. Two hundred forty jelly beans, each of volume $0.5\,\text{cm}^3$, are placed inside a cylindrical jar of radius $5\,\text{cm}$ and height $6/\pi\,\text{cm}$. What is the maximum number of milliliters of water that may be poured into the jar along with the jelly beans? ($1\,\text{cm}^3 = 1\,\text{mL}$)

(a) 20
(b) 40
(c) 60
(d) 80
(e) None of the above

6. Jay writes down a list of positive integers and multiplies them all together, and he obtains result of 168,894. The prime number 853 is a factor of 168,894. Which of the following statements, if any, may be deduced from this information?

I. The list must contain more than two numbers.
II. The list must contain at least one member which is greater than 500.
III. The list must contain the number 2.

(a) I only
(b) II only
(c) I and III
(d) I and II
(e) None of the statements I, II, or III may be deduced.

7. Point A has x-coordinate 2 and lies on the graph of $y = x^2$. Point B has an x-coordinate of 2.1 and also lies on the graph of $y = x^2$. Find the slope of a line that passes through the points A and B.

(a) 3.9
(b) 4.0
(c) 4.1
(d) 4.2
(e) None of the above

8. Tara remembers her friend's phone number as 4439754456, but when she calls it she finds out this is the wrong number. She is sure that she switched two adjacent digits around, but she doesn't know which ones. What is the maximum number of additional times she might have to dial a wrong number before getting her friend?

(a) 4
(b) 5
(c) 6
(d) 7
(e) 8

9. Given that

$$\frac{2}{3} < \frac{a}{b} < \frac{3}{4}$$

for positive integers a and b, what is the minimum possible value of b?

(a) 10
(b) 9
(c) 7
(d) 5
(e) None of the above

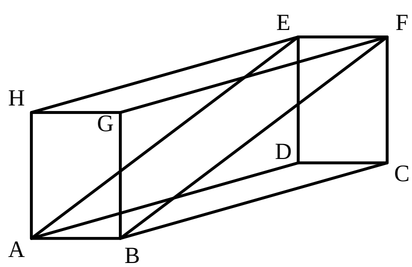
10. Two-thirds of Andrea's coins are pennies. If she were given ten more pennies, three-fourths of her coins would be pennies. What is the greatest amount of money Andrea could have in coins? (Assume that she does not have any coins that are not pennies, nickels, dimes, or quarters).

(a) \$2.70
(b) \$2.90
(c) \$3.10
(d) \$3.30
(e) \$4.05

11. Addie is reading *Twenty Thousand Leagues Under the Sea*. Based on the total number of pages in the book, she calculates that it will take her 20–22 days reading at a pace of 30–33 pages per day. How many pages are in her book?

(a) 600
(b) 620
(c) 640
(d) 660
(e) 726

12. The $5 \times 10 \times 24$ rectangular prism shown, with $EF = 5$, $FC = 10$, and $BC = 24$, is cut into two pieces across the plane $ABFE$. Find the surface area of one of the two resulting solids.



(a) 480
(b) 540
(c) 560
(d) 600
(e) None of the above

13. Let $r = \dfrac{3 \cdot 71 \cdot 109}{41 \cdot 37} - \dfrac{109 \cdot 9 \cdot 223}{41 \cdot 37 \cdot 10}$. Then

(a) r is irrational
(b) r is positive
(c) $r = 0$
(d) r is negative
(e) None of the above

14. Jacob has three drawers, a top drawer, a middle drawer, and a bottom drawer, and each of the drawers is associated with either shirts, socks, or shorts. No type of clothing is associated with two different drawers. The socks are not in the top drawer, the shorts are not in the middle drawer, and the shirts are not in the bottom drawer. How many of the following statements are true?

I. The shorts are in the top drawer
II. The shirts are in the top drawer
III. The shirts are in the middle drawer
IV. The socks are in the middle drawer

(a) exactly 2
(b) exactly 1
(c) exactly 0
(d) either 1 or 2
(e) either 0 or 1

15. A *proper factor* of an integer m is a number k such that m/k is an integer and $k \neq 1$ and $k \neq m$. Define the function $L(n)$ to be the largest proper factor of n. For example, $L(6) = 3$. For how many positive integers n is $L(n) = 7$?

(a) 4
(b) 3
(c) 2
(d) 1
(e) infinitely many

16. Last season, the Trinkenville baseball team had one win and nine losses on July 3, but by July 17 they had won exactly five-ninths of their games. What is the minimum total number of games Trinkenville could have played last season by July 17?

(a) 18
(b) 21
(c) 27
(d) 36
(e) None of the above

17. Compute the difference

$$982436 \cdot 31175 - 982435 \cdot 31174$$

(a) 1013210
(b) 1013310
(c) 1023210
(d) 1023310
(e) None of the above

18. On a certain planet, exactly 1/50 of pregnancies result in the birth of twins, and the remaining pregnancies result in the birth of singletons. What is the probability that a randomly selected person on this planet has a twin?

(a) 2%
(b) 4%
(c) strictly greater than 4%
(d) strictly between 2% and 4%
(e) None of the above

19. Let $\mathscr{S}$ denote the set of points in three-dimension Euclidean space such that for any point $P \in \mathscr{S}$, the distance from P to each of three certain non-collinear points A, B, and C is the same. For different arrangements of the points A, B, and C, the set $\mathscr{S}$ is

(a) always a sphere
(b) always a plane
(c) sometimes a line and sometimes empty
(d) sometimes a line and sometimes a point
(e) always a line

20. For nonnegative integer values of n, let $f(n)$ be the fewest number of American coins (quarters, nickels, dimes, pennies) that are required to make n cents. For example, $f(9) = 5$ and $f(56) = 4$. For how many positive integers n is $f(n) = 6$?

(a) 25
(b) 27
(c) 29
(d) 31
(e) None of the above

21. For θ in the first quadrant, the trigonometric expression $\sqrt{1+\sin(2\theta)}$ may be written uniquely in the form $a\cos(b\pi-\theta)$, where a and b are real numbers with $0 \le b \le 1/2$. Find a^2b^2.

(a) 2
(b) 1
(c) 1/2
(d) 1/4
(e) 1/8

22. Suppose that $f(x)$ is a polynomial such that when $f(x)$ is divided by x^2+x+1, the quotient is $x^{101}-x^{99}+6x^{45}$, and the remainder is 0. Let $r(x)$ be the remainder when $f(x)$ is divided by x^3-1. Find $r(2)$.

(a) 6
(b) 30
(c) 42
(d) 49
(e) None of the above

23. Given that one of the answer choices is correct, which of the following is equal to $\sqrt[5]{28629151^2}$?

(a) 881
(b) 901
(c) 941
(d) 961
(e) 1021

24. For pairs of real numbers (x_1,x_2) with $-1 \le x_2 \le 1$, find the minimum value of the expression

$$(x_1-x_2)^2+\left(10-2x_1-\sqrt{1-x_2^2}\right)^2.$$

(a) $25-2\sqrt{10}$
(b) $25-4\sqrt{6}$
(c) $21-4\sqrt{5}$
(d) $19-6\sqrt{2}$
(e) None of the above

25. There exist two real solutions x_1 and x_2 of the equation

$$\left(\sqrt{3}-x\right)\left(1+\sqrt{x^2+1}\right)=x\left(1+x\sqrt{3}\right).$$

For any real t, let $\operatorname{Arctan} t$ be the number θ such that $-\pi/2<\theta<\pi/2$ and $\tan\theta = t$. To the nearest thousandth, find $\operatorname{Arctan} x_1 + \operatorname{Arctan} x_2$.

(a) -0.698
(b) 0.573
(c) -0.106
(d) 0.391
(e) None of the above