University of Mississippi

Individual Competition

11th Annual High School Mathematics Contest

1. A subset of the integers $1, 2, \ldots, 100$ has the property that none of its members is 5 times another. What is the largest number of members such a subset can have?

(a) 72
(b) 77
(c) 84
(d) 85
(e) 86

2. The cubic equation $x^3 + 2x^2 - x + 1 = 0$ has three roots, x_1, x_2 and x_3. What is $x_1 \cdot x_2 \cdot x_3$?

(a) -1
(b) 0
(c) 1
(d) 2
(e) None of the above

3. What is the last digit of 3^{2015}?

(a) 1
(b) 3
(c) 5
(d) 7
(e) 9

4. Each interior angle of a regular n-sided polygon is $144°$. What is n, the number of sides?

(a) 5
(b) 6
(c) 8
(d) 10
(e) 12

5. Alice, Bob, and Clarissa split \$5000 among themselves to be invested in different ways. Each begins with a different amount. At the end of one year, they have a total of \$7500. Bob and Clarissa have both doubled their money, whereas Alice has lost \$1000. How much money did Alice start with?

(a) \$ 850
(b) \$ 1000
(c) \$1500
(d) \$ 2100
(e) Inadequate information

6. When the expression $(5x - 6)^{12}$ is expanded, what is the sum of the coefficients?

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2

7. Merlin has a magic coin changing machine. When he puts in a quarter, it returns five nickels; when he puts in a nickel, it returns five pennies; and when he puts in a penny, it returns five quarters. Merlin starts with just one penny. Which of the following amounts could Merlin have after using the machine repeatedly?

(a) \$ 2.17
(b) \$ 3.73
(c) \$ 3.75
(d) \$ 4.12
(e) \$ 5.20

8. A regular n-sided polygon has 2015 diagonals. What is the value of n?
(For a polygon, a diagonal is any straight line segment between two vertices that isn't a side. For example, a pentagon has 5 diagonals.)

(a) 55
(b) 62
(c) 65
(d) 70
(e) Does not exist

9. Two positive numbers a and b satisfy $a + b = \sqrt{52}$ and $a^2 + b^2 = 32$. What is the value of $\log_{10} a + \log_{10} b$?

(a) -1
(b) $\frac{1}{2} \log_{10} 20$
(c) 1
(d) $\log_{10} 20$
(e) 2

10. Jennifer has 64 fair coins. She tosses all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are heads at the end?

(a) 32
(b) 48
(c) 52
(d) 56
(e) 64

11. Larry can make one batch of 3 cookies every 8 minutes. Mary can make one batch of 5 cookies every 12 minutes. How long will it take them to make a combined total of 60 cookies?

(a) 1 h 5 min
(b) 1 h 12 min
(c) 1 h 20 min
(d) 1 h 24 min
(e) 1 h 30 min

12. What is the greatest common divisor of 14! and of $14!+11^2$. ($14! = 14 \cdot 13 \cdot \ldots 3 \cdot 2 \cdot 1$)
(a) 1
(b) 11
(c) 14
(d) 11^2
(e) 11!

13. It would take Tom 3 hours to paint the fence and Huck would take 5 hours to paint it. They work on it together. How long does it take them to paint the fence? (Assume they don't get in each other's way.)
(a) 1 hour
(b) $\frac{15}{8}$ hours
(c) 2 hours
(d) $\frac{17}{8}$ hours
(e) $\frac{9}{4}$ hours

14. Janel and Alex play a game where they toss a single coin. Janel wins if the coin lands on heads and Alex wins if it lands on tails. They play 10 times. What is the probability that they have won an equal number of times?
(a) 2^{-10}
(b) 2^{-11}
(c) $90 \cdot 2^{-11}$
(d) $10! \cdot 2^{-10}/(5!)^2$
(e) $\frac{1}{2}$

15. Mr. Earl E. Bird leaves home every day at 8:00 AM to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

(a) 45
(b) 48
(c) 50
(d) 55
(e) 58

16. What does

$$\sqrt[3]{x\sqrt[3]{x\sqrt[3]{x\sqrt[3]{\ldots}}}}$$

simplify to (for positive x)?

(a) $\sqrt[3]{x}$
(b) $\sqrt{x}$
(c) x
(d) x^2
(e) x^3

17. Let r and s be the two roots of $x^2 - 4x + 1 = 0$. What is $r^2 + s^2$?

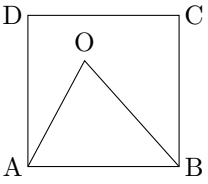
(a) -12
(b) 0
(c) 12
(d) 14
(e) $12\sqrt{3}$

18. What is this sum equal to

$$\sum_{n=1}^{99} \frac{1}{n(n+1)} \coloneqq \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{99\cdot 100}?$$

(a) 99/100
(b) 1
(c) 100/99
(d) 2
(e) None of the above

19. A point O is chosen at random in the square ABCD. What is the probability that the angle $\angle AOB$ is larger than $90°$?



(a) $\frac{1}{2}$
(b) $\frac{1}{3}$
(c) $\frac{3}{16}$
(d) $\frac{3\pi}{16}$
(e) $\frac{\pi}{8}$

20. If $a_{n+1} = \frac{a_n}{4^n}$, and $a_1 = 1$, what is a_n?

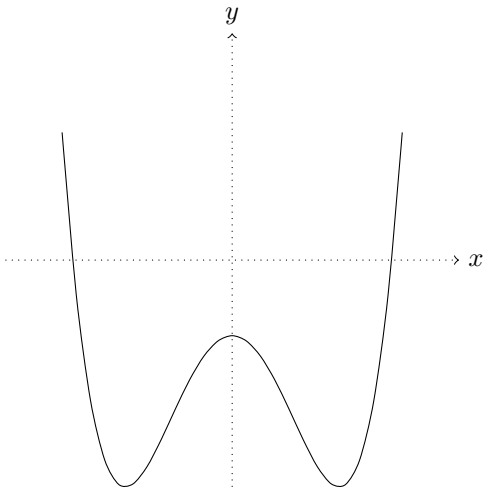
(a) $\frac{1}{4^{n(n+1)}}$
(b) $\frac{1}{2^{n(n+1)}}$
(c) 1
(d) $2^{(1-n)n}$
(e) $4^{(1-n)n}$

21. In triangle ABC, $\sin A - \sin B = \frac{\sqrt{2}}{2}$ and $\cos A + \cos B = \frac{\sqrt{2}}{2}$. What is the angle C?

(a) $30°$
(b) $45°$
(c) $60°$
(d) $90°$
(e) $120°$

22. The following is the graph of some function

$$f(x) = ax^4 + bx^2 + c.$$



What are the signs of the constants a, b and c?

(a) $a > 0$, $b > 0$, $c > 0$
(b) $a > 0$, $b < 0$, $c > 0$
(c) $a > 0$, $b < 0$, $c < 0$
(d) $a > 0$, $b > 0$, $c < 0$
(e) $a < 0$, $b > 0$, $c < 0$

23. What is the greatest common divisor of

$$2^{2016} - 1 \text{ and } 2^{2014} - 1?$$

(a) 1
(b) 2
(c) 3
(d) 4
(e) 9